A Denotational Approach to Release/Acquire Concurrency

We want to report on recent and ongoing work into the denotational semantics of shared-state concurrent programming languages with Brookes-style trace semantics. Most of the talk would cover our trace semantics for the Release/Acquire (RA) memory model, a fragment of the C/C++ standard. Developing this semantics required us to re-think the interpretation of Brookes trace-sets, moving away from interrupted executions and towards a game-like/rely-guarantee-like intuition about the interaction of the program with its environment. We would like to present to the GALOP community these results and, time permitting, ongoing work about more general trace semantics for shared state, to facilitate discussion and further directions.

For shared-memory concurrent programming, the seminal work of Brookes [1996] defined a denotational semantics, where the denotation $[\![M]\!]$ is a set of totally ordered traces that consist of sequences of pairs of memory snapshots $\langle \mu_0, \varrho_0 \rangle \dots \langle \mu_n, \varrho_n \rangle$. Each sequence represents a behavior that the program fragment M may exhibit. In every pair $\langle \mu, \varrho \rangle$ in a trace, μ is the snapshot that M relies on the environment to provide; and ϱ is the snapshot that M guarantees to provide in return. The gaps between pairs represent possible interference by the environment. Working under the assumption of preemptive scheduling—imposing no restrictions on the interleaving of steps of execution between parallel threads—denotations are closed under the following two trace-rewriting operations which maintain the representation of possible behavior. *Stutter* adds a transition of the form $\langle \mu, \mu \rangle$ anywhere in the trace; a program fragment can always guarantee no changes between its actions. *Mumble* combines a couple of subsequent transitions of the form $\langle \mu, \varrho \rangle \langle \varrho, \theta \rangle$ into a single transition $\langle \mu, \theta \rangle$ anywhere in the trace; a program fragment can always rely on its own guarantees in the absence of observable interference from the environment.

A *memory model* describes how memory access by concurrently running threads is handled through a program's routine. Brookes established the adequacy of the trace-based denotational semantics w.r.t. the strongest operational semantics of shared-memory concurrent programs, known as *sequential consistency* (SC), where every memory access happens instantaneously and immediately affects all concurrent threads. Jagadeesan et al. [2012] closely followed Brookes to define denotational semantics for x86-TSO [Owens et al. 2009; Pulte et al. 2018]. Other weak memory models, in particular, models of *programming languages*, and *non-multi-copy-atomic* models, where writes can be observed by different threads in different orders, have so far been out of reach of Brookes's totally ordered traces, and were only captured by much more sophisticated models based on *partial orders* [Castellan 2016; Dodds et al. 2018; Jagadeesan et al. 2020; Jeffrey et al. 2022; Kavanagh and Brookes 2018; Paviotti et al. 2020]. In this work we target the Release/Acquire memory model. This model, obtained by restricting the C/C++11 memory model [Batty et al. 2011] to release/acquire atomics, is a well-studied fundamental memory model weaker than x86-TSO, which, roughly speaking, ensures "causal consistency" together with "per-location-SC" and "RMW (read-modify-write) atomicity" [Lahav 2019; Lahav et al. 2016]. These guarantees make RA sufficiently strong for implementing common synchronization idioms.

Our first contribution is a Brookes-style denotational semantics for RA. As Brookes's traces are totally ordered, this result may seem counterintuitive. The standard semantics for RA is a declarative (a.k.a. axiomatic) memory model, in the form of acyclicity consistency constraints over partially ordered candidate execution graphs. Since these partial orders are not totally ordered, one might expect that Brookes's traces are insufficient. Nevertheless, our first key observation is that an *operational* presentation of RA as an interleaving semantics of a weak memory system lends itself to Brookes-style semantics. We develop a notion of traces compatible with Kang et al.'s "view-based" machine [2017], an operational semantics that is equivalent to RA's declarative formulation. There, a threads writes by adding a message to memory. Once added, a message is never removed or modified. The memory associates a timeline to each location, on which messages are placed, each occupying a segment. Messages cannot overlap, but they can be placed adjacently. If the message originated in an RMW, the written message must be placed adjacently to the read message. This blocks another RMW from doing the same, thus enforcing atomicity. Each thread maintains a view that determines, for each location, what is the latest messages of which it was made aware on the timeline. A thread cannot read nor write messages prepending its last known message. To enforce causal consistency, each message records the view of the thread that wrote it. When a thread reads a message, it inherits its view, possibly making it aware of later messages.

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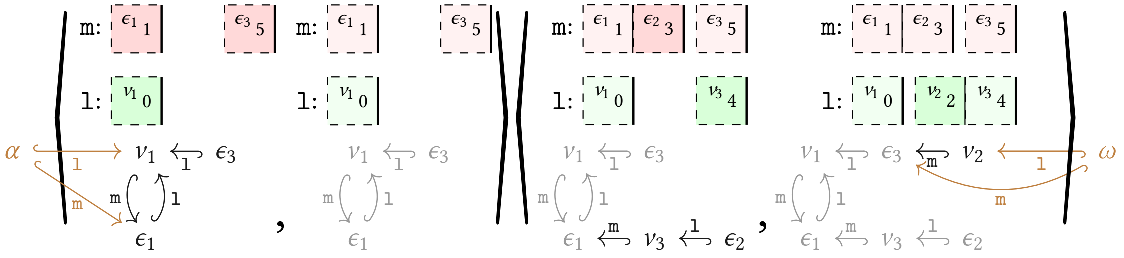
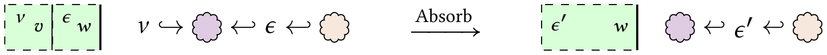
Below we illustrate an example trace, in the setting of two memory locations `l` and `m`. The trace has two pairs of memory snapshots, illustrated using two diagrams. Top: shows the messages per location as (possibly adjacent) segments on the location's timeline. Bottom: shows the graph structure induced by the views within messages. The only message added within a transition is ν_2, and thus it is the only *local* message: one that the program fragment guarantees to provide, rather than relies on receiving. The trace also has an initial view α and a final view ω, illustrated by showing the messages to which they point.



In the future, we hope to bring in nominal techniques to account for timestamps in a more principled manner. In particular, we would like to investigate the possibility of using a Fraïssé limit [Bojanczyk et al. 2012; Fraïssé 1986] for comparing names for (i) equality, (ii) order, and (iii) adjacency.

We prove several results about our denotational semantics. Soundness: for every interrupted execution there is a corresponding single-transition trace in the denotation. Fundamental Lemma: for every trace in the denotation there exists an interrupted execution of the program fragment exhibiting a related behavior. Adequacy: denotational approximation implies contextual refinement. An immediate practical application of adequacy is the ability to provide *local* formal justifications of program transformations, such as those performed by optimizing compilers. Formally justifying these transformations without the local analysis that denotational semantics provides is non-trivial [Dodds et al. 2018; Vafeiadis et al. 2015].

An important aspect of denotational semantics is its abstraction. As an external measure, we verify that our adequate semantics validates various transformations/optimizations: standard and structural transformations; algebraic laws of parallel programming; and all known thread-local RA-valid compiler transformations involving atomic RA memory accesses. This level of abstraction is achieved thanks to our denotations being closed not only under analogs to Brookes's stutter and mumble, but also several RA-specific operations. This allows us to relate programs which would naively correspond to rather different sets of traces. For example, we have the *Absorb* rewrite rule, which combines two adjacent local messages added within the same transition. Below we sketch how it modifies memory snapshots:



Figuratively, the preceding message ν is "absorbed" by the successor ϵ, thus becoming ϵ'. Nothing must point to the preceding message ν, so as to not leave dangling names. We use this rule when validating transformations which eliminate a write that is followed by another, such as `l` := v ; `l` := w $\twoheadrightarrow$ `l` := w.

Our second contribution is to connect the core semantics of parallel programming languages exhibiting weak behaviors to the more standard semantic account for sequential programming languages. Brookes presented his semantics for a simple imperative WHILE language, but Benton et al. [2016]; Dvir et al. [2022] later extended it to higher-order languages using Moggi's monad-based approach [1991].

A denotational semantics given in this monadic style comes ready-made with a rich semantic toolkit for program denotation [Benton et al. 2000], transformations [Benton et al. 2014, 2007, 2009; Benton and Leperchey 2005; Hofmann 2008], reasoning [Aguirre et al. 2022; Maillard et al. 2019], etc. We want to challenge, compare, and reuse this diverse toolkit in the concurrent setting. As a yardstick to the applicability of the monadic toolkit, we develop our semantics for a *higher-order functional language* with a general, first-class parallel composition operator. This is in contrast to most of the weak memory models research which employs imperative languages and assumes a single top-level parallel composition, but more in line with game models for concurrency [e.g. Castellan et al. 2017]. This puts weak memory models, which often require bespoke and highly specialized presentations, on a similar footing to many other programming effects.

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APPENDIX

The table below summarizes the transformations that we have validated using our denotational semantics. Some are given first using the general **rmw** construct, then specialized to loads (?) and well-known RMWs (CAS, FAA, XCHG). When a non-trivial closure rule (Ab, Ti, Di) is used for the denotational justification it appears above the symbol ↠.

Generalized Sequencing

$$(\mathbf{let}\,\mathrm{x} = M_1\,\mathbf{in}\,M_2) \parallel (\mathbf{let}\,\mathrm{y} = N_1\,\mathbf{in}\,N_2) \twoheadrightarrow \mathbf{match}\,M_1 \parallel N_1\ \mathbf{with}\,\langle \mathrm{x}, \mathrm{y}\rangle.\,M_2 \parallel N_2$$

Sequencing $M \parallel N \twoheadrightarrow \langle M, N\rangle$

Irrelevant Read Introduction $\langle\rangle \twoheadrightarrow \ell?\,;\langle\rangle$

Irrelevant Read Elimination $\ell?\,;\langle\rangle \twoheadrightarrow \langle\rangle$

Write-Write Elimination

$$\ell := w\,;\ell := v \ \overset{\mathsf{Ab}}{\twoheadrightarrow}\ \ell := v$$

Write-Read Deorder $(\ell \neq \ell')$

$$\left\langle \ell := v, \ell'?\right\rangle \ \overset{\mathsf{Ti}}{\twoheadrightarrow}\ \ell := v \parallel \ell'?$$

RMW Expansion $(\varphi_{\vec{v}} \leq \psi_{\vec{w}})$

$$\mathbf{rmw}_{\varphi}\,(\ell;\vec{v}) \ \overset{\mathsf{Di}}{\twoheadrightarrow}\ \mathbf{rmw}_{\psi}\,(\ell;\vec{w})$$

$$\ell? \ \overset{\mathsf{Di}}{\twoheadrightarrow}\ \mathrm{CAS}\,(\ell, v, v)$$

$$\mathrm{CAS}\,(\ell, v, v) \ \overset{\mathsf{Di}}{\twoheadrightarrow}\ \mathrm{FAA}\,(\ell, 0)$$

Atomic Store

$$\ell := v \ \twoheadrightarrow\ \mathrm{XCHG}\,(\ell, v)\,;\langle\rangle$$

Symmetric-Monoidal Laws, e.g.

$$M \parallel N \ \twoheadrightarrow\ \mathbf{match}\,N \parallel M\ \mathbf{with}\,\langle \mathrm{x}, \mathrm{y}\rangle.\ \langle \mathrm{y}, \mathrm{x}\rangle$$

Write-RMW Elimination

$$\ell := v\,;\mathbf{rmw}_{\varphi}\,(\ell;\vec{w}) \ \overset{\mathsf{Ab}}{\twoheadrightarrow}\ \ell := \varphi^{\mathrm{id}}_{\vec{w}}v\,;v$$

$$\ell := v\,;\ell? \ \twoheadrightarrow\ \ell := v\,;v$$

$$\ell := v\,;\mathrm{CAS}\,(\ell, v, u) \ \overset{\mathsf{Ab}}{\twoheadrightarrow}\ \ell := u\,;v$$

$$\ell := v\,;\mathrm{CAS}\,(\ell, w, u) \ \twoheadrightarrow\ \ell := v\,;v \qquad (v \neq w)$$

$$\ell := v\,;\mathrm{FAA}\,(\ell, w) \ \overset{\mathsf{Ab}}{\twoheadrightarrow}\ \ell := v + w\,;v$$

$$\ell := v\,;\mathrm{XCHG}\,(\ell, w) \ \overset{\mathsf{Ab}}{\twoheadrightarrow}\ \ell := w\,;v$$

RMW-Write Elimination $(\mathrm{dom}\,\psi_{\vec{w}} \supseteq \mathrm{dom}\,\varphi_{\vec{u}})$

$$\begin{aligned}&\mathbf{let}\,\mathrm{x} = \mathbf{rmw}_{\varphi}\,(\ell;\vec{u})\ \mathbf{in}\\ &\quad \mathbf{match}\,(\psi_{\vec{w}})\,\mathrm{x}\,\mathbf{with}\\ &\quad \{\iota_{\perp}\,\_.\mathrm{x} \mid \iota_{\top}\,v.\ell := v\,;\mathrm{x}\} \ \overset{\mathsf{Ab}}{\twoheadrightarrow}\ \mathbf{rmw}_{\psi}\,(\ell;\vec{w})\end{aligned}$$

$$\begin{aligned}&\mathbf{let}\,\mathrm{x} = \ell?\,\mathbf{in}\,(\mathbf{if}\,\mathrm{x} = v\\ &\quad \mathbf{then}\,\ell := w\,\mathbf{else}\,\langle\rangle\,)\,;\mathrm{x} \ \twoheadrightarrow\ \mathrm{CAS}\,(\ell, v, w)\end{aligned}$$

$$\mathbf{let}\,\mathrm{x} = \ell?\,\mathbf{in}\,\ell := \mathrm{x} + v\,;\mathrm{x} \ \twoheadrightarrow\ \mathrm{FAA}\,(\ell, v)$$

$$\mathbf{let}\,\mathrm{x} = \ell?\,\mathbf{in}\,\ell := v\,;\mathrm{x} \ \twoheadrightarrow\ \mathrm{XCHG}\,(\ell, v)$$

RMW-RMW Elimination $\left\langle \mathbf{rmw}_{\varphi}\,(\ell;\vec{v}), \mathbf{rmw}_{\psi}\,(\ell;\vec{w})\right\rangle \overset{\mathsf{Ab}}{\twoheadrightarrow} \mathbf{let}\,\mathrm{x} = \mathbf{rmw}_{\zeta}\,(\ell;\vec{u})\ \mathbf{in}\ \left\langle \mathrm{x}, \varphi^{\mathrm{id}}_{\vec{v}}\mathrm{x}\right\rangle \quad (\zeta_{\vec{u}} = \psi_{\vec{w}} \circ^{\mathrm{id}} \varphi_{\vec{v}})$

$$\langle \ell?, \ell?\rangle \twoheadrightarrow \mathbf{let}\,\mathrm{x} = \ell?\,\mathbf{in}\,\langle \mathrm{x}, \mathrm{x}\rangle \qquad \langle \mathrm{FAA}\,(\ell, v), \mathrm{FAA}\,(\ell, w)\rangle \twoheadrightarrow \mathbf{let}\,\mathrm{x} = \mathrm{FAA}\,(\ell, v + w)\ \mathbf{in}\ \langle \mathrm{x}, \mathrm{x} + v\rangle$$

$$\langle \ell?, \mathrm{CAS}\,(\ell, v, w)\rangle \twoheadrightarrow \mathbf{let}\,\mathrm{x} = \mathrm{CAS}\,(\ell, v, w)\ \mathbf{in}\ \langle \mathrm{x}, \mathrm{x}\rangle \qquad \langle \mathrm{XCHG}\,(\ell, w), \ell?\rangle \twoheadrightarrow \mathbf{let}\,\mathrm{x} = \mathrm{XCHG}\,(\ell, w)\ \mathbf{in}\ \langle \mathrm{x}, w\rangle$$